

Exponential Models

1.

The value of a car is modelled by the formula

$$V = 20\,000e^{-\frac{t}{12}}$$

where V is the value in £s and t is its age in years from new.

- a State its value when new.
- b Find its value (to the nearest £) after 4 years.
- c Sketch the graph of V against t .

2.

The population of a country is modelled using the formula

$$P = 20 + 10e^{\frac{t}{50}}$$

where P is the population in thousands and t is the time in years after the year 2000.

- a State the population in the year 2000.
- b Use the model to predict the population in the year 2030.
- c Sketch the graph of P against t for the years 2000 to 2100.
- d Do you think that it would be valid to use this model to predict the population in the year 2500? Explain your answer.

3.

The number of people infected with a disease is modelled by the formula

$$N = 300 - 100e^{-0.5t}$$

where N is the number of people infected with the disease and t is the time in years after detection.

- a How many people were first diagnosed with the disease?
- b What is the long term prediction of how this disease will spread?
- c Sketch the graph of N against t for $t > 0$.

4.

The number of rabbits, R , in a population after m months is modelled by the formula

$$R = 12e^{0.2m}$$

- a Use this model to estimate the number of rabbits after
 - i 1 month
 - ii 1 year
- b Interpret the meaning of the constant 12 in this model.
- c Show that after 6 months, the rabbit population is increasing by almost 8 rabbits per month.
- d Suggest one reason why this model will stop giving valid results for large enough values of t .

Problem-solving

Your answer to part **b** must refer to the context of the model.

5.

On Earth, the atmospheric pressure, p , in bars can be modelled approximately by the formula $p = e^{-0.13h}$ where h is the height above sea level in kilometres.

- a** Use this model to estimate the pressure at the top of Mount Rainier, which has an altitude of 4.394 km. **(1 mark)**
- b** Demonstrate that $\frac{dp}{dh} = kp$ where k is a constant to be found. **(2 marks)**
- c** Interpret the significance of the sign of k in part **b**. **(1 mark)**
- d** This model predicts that the atmospheric pressure will change by $s\%$ for every kilometre gained in height. Calculate the value of s . **(3 marks)**

6.

Nigel has bought a tractor for £20 000. He wants to model the depreciation of the value of his tractor, £ T , in t years. His friend suggests two models:

Model 1: $T = 20\,000e^{-0.24t}$

Model 2: $T = 19\,000e^{-0.255t} + 1000$

- a** Use both models to predict the value of the tractor after one year.
Compare your results. **(2 marks)**
- b** Use both models to predict the value of the tractor after ten years.
Compare your results. **(2 marks)**
- c** Sketch a graph of T against t for both models. **(2 marks)**
- d** Interpret the meaning of the 1000 in model 2, and suggest why this might make model 2 more realistic. **(1 mark)**